

# Spanning Tree



- Any tree consisting solely of edges in  $G$  and including all vertices in  $G$  is called a *spanning tree*.
- Spanning tree can be obtained by using either a depth-first or a breath-first search.
- When a nontree edge  $(v, w)$  is introduced into any spanning tree  $T$ , a cycle is formed.
- A spanning tree is a minimal subgraph,  $G'$ , of  $G$  such that  $V(G') = V(G)$ , and  $G'$  is connected. (Minimal subgraph is defined as one with the fewest number of edges).
- Any connected graph with  $n$  vertices must have at least  $n-1$  edges, and all connected graphs with  $n-1$  edges are trees. Therefore, a spanning tree has  $n-1$  edges.

# Spanning Tree



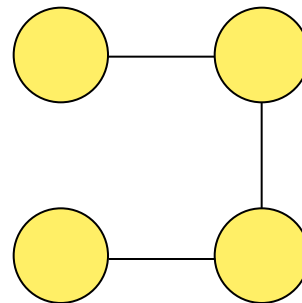
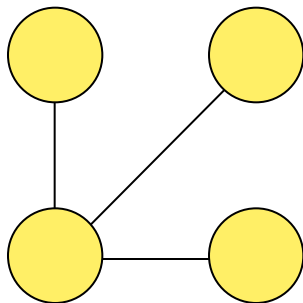
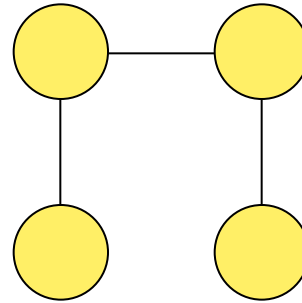
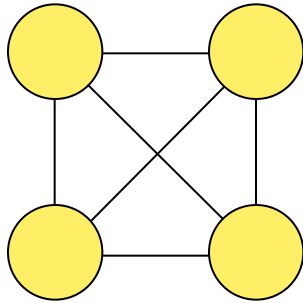
When  $G$  is connected, DFS or BFS is applied, then the edges is partitioned into  $T$  and  $N$

$T$ : edges used during traversal, also called tree edges

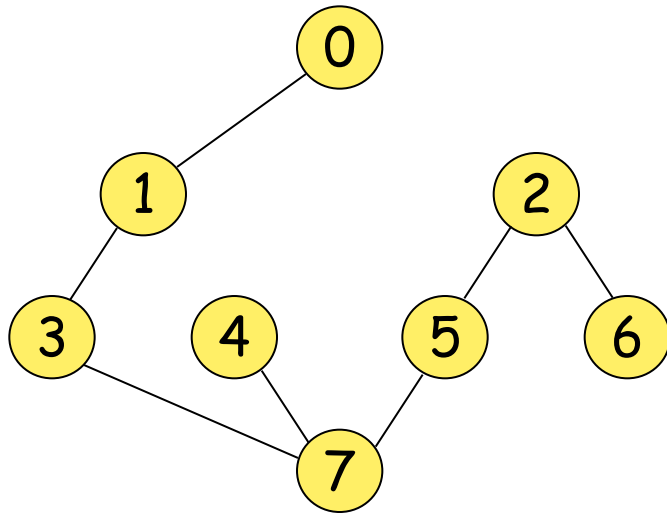
$N$ : nontree edges

Spanning tree: all vertices +  $T$

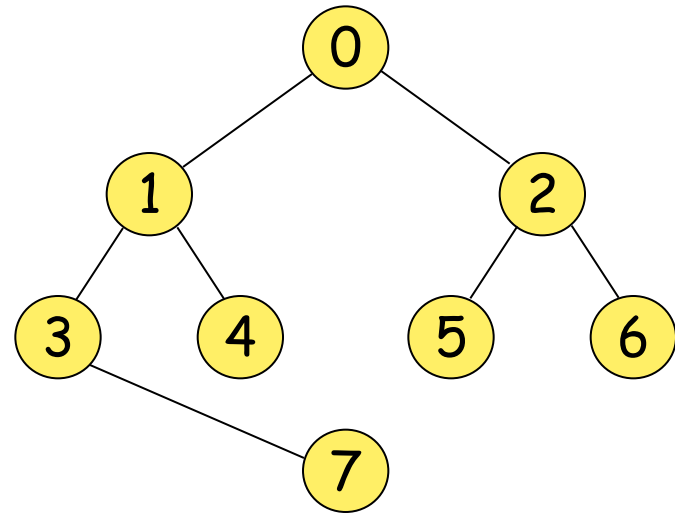
# A Complete Graph and Three of Its Spanning Trees



# Depth-First and Breadth-First Spanning Trees



(a) DFS (0) spanning tree



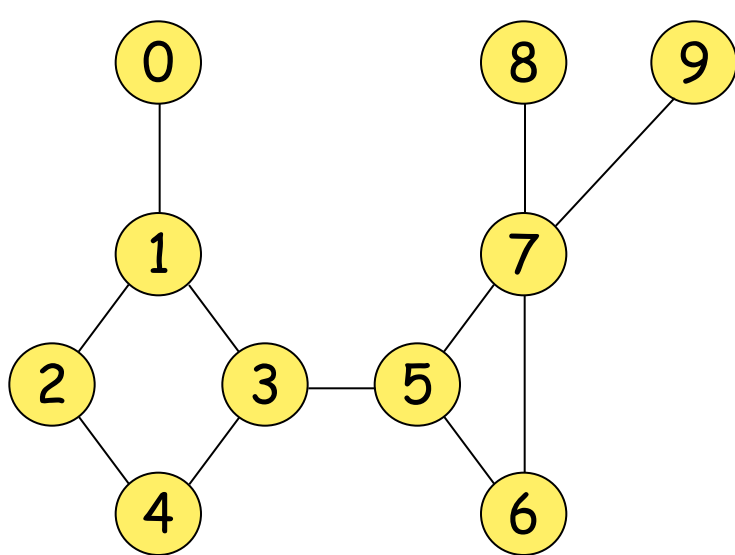
(b) BFS (0) spanning tree

# Biconnected Components

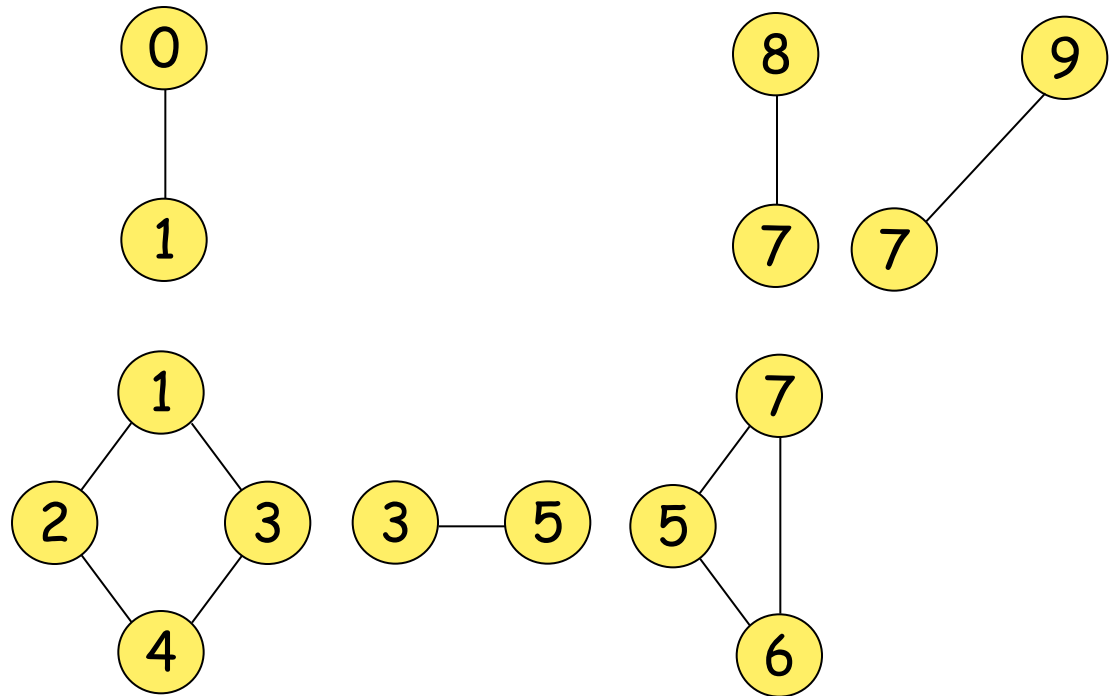


- Definition: A vertex  $v$  of  $G$  is an **articulation point** iff the deletion of  $v$ , together with the deletion of all edges incident to  $v$ , leaves behind a graph that has **at least two connected components**.
- Definition: A biconnected graph is a connected graph that has **no articulation points**.
- Definition: A **biconnected** component of a connected graph  $G$  is a **maximal biconnected** subgraph  $H$  of  $G$ . By maximal, we mean that  $G$  contains no other subgraph that is both biconnected and properly contains  $H$ .

# A Connected Graph and Its Biconnected Components



(a) A connected graph



(b) Its biconnected components

Maximal without articulation point

# Efficiency of Algorithm



- Algorithm efficiency is equal to the function of number of elements to be processed.
- We must know efficiency of loop

# Linear loop



- Example

$i=1$

Loop( $i \leq 10$ )

...

$i=i+1$



# Logarithmic loop



Example 1

$i=1$

Loop( $i < 1000$ )

....

$i=i*2$

Example 2

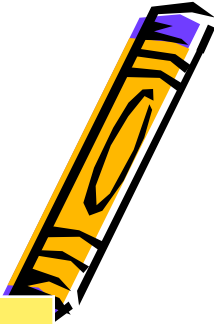
$i=1000$

Loop( $i >= 1$ )

...

$i=i/2$

# Logarithmic loop (continued)



Iteration	Value of i(multiplication)	Iteration	Value of i(Division)
1	1	1	1000
2	2	2	500
3	4	3	250
4	8	4	125
5	16	5	62
6	32	6	31
7	64	7	15
8	128	8	7
9	256	9	3
10	512	10	1
Exit	1024	Exit	0

# Nested loop



- Iteration=Outer loop iteration\*  
Inner loop iteration
- Three types of nested loop
  - Linear Logarithmic
  - Dependent Quadratic
  - Quadratic

# Linear logarithmic



- Example

$i=1$

loop( $i \leq 10$ )

$j=1$

    loop( $j \leq 10$ )

    ...

$j=j*2$

$i=i*2$

# Dependent Quadratic



- Example

$i=1$

loop( $i \leq 10$ )

$j=1$

loop( $j \leq i$ )

...

$j=j+1$

$i=i+1$

# Quadratic



- Example

$i=1$

loop( $i \leq 10$ )

$j=1$

loop( $j \leq 10$ )

...

$j=j+1$

$i=i+1$

# Example1



Statement	s/e	frequency	Total steps
Algorithm Sum(a, n)	0	-	0
{	0	-	0
s=0.0;	1	1	1
for i:=1 to n do	1	n+1	n+1
s=s +a[i];	1	n	n
return s;	1	1	1
}	0	-	0
Total			2n+3

# Application



- Network flow
- Bridge Block problem
- Cluster



# Scope of research



- **Rapid protein side-chain prediction**

# Assignment



Q.1) What is bi-connected graph? Give an example of the bi-connected component.

Q.2) What is articulation point?

Q.3) What is efficiency of following algorithm

Unsigned int fibonacci (Unsigned int n)

```
{  
    int previous=-1;  
    int result=1;  
    for(unsigned int i=0;i<=n;++i)  
    {  
        int sum=result+previous;  
        previous=result;  
        result=sum;  
    }  
    return sum;  
}
```